

# On Open Inflation, the string theory landscape and the low CMB quadrupole

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Cosmologists have embraced a particular ad hoc formula for the primordial power spectrum from inflation for universes with  $\Omega_0 < 1$ . However, the so-called “Open Inflation” models, which are attracting renewed interest in the context of the “string theory landscape” give a different result, and offer a more fully developed picture of the cosmology and fundamental physics basis for inflation with  $\Omega_0 < 1$ . The Open Inflation power spectrum depends not only on  $\Omega_0$ , but on the parameters of the effective fields that drive the universe *before* the Big Bang (in “another part of the landscape”). This paper considers the search for features in CMB temperature anisotropy data that might reflect a primordial spectrum of the Open Inflation form. We ask whether this search could teach us about high energy physics that described the universe before the onset of the Big Bang, and perhaps even account for the low CMB quadrupole. Unfortunately our conclusion is that the specific features we consider are unobservable even with future experiments although we note a possible loophole connected with our use of the thin wall approximation.

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**Note Added:** Since this paper was completed we became aware of a large body of existing literature which treats the problem of perturbations in Open Universe models with a much greater degree of sophistication than we do here (including working away from the thin wall limit). See [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] and references therein.

An up-to-date treatment of the important questions raised in this paper (about the possible universality of open inflation in the string theory landscaped and resulting observational signatures) requires the application of these more sophisticated methods and results, a process we are now undertaking. We apologize to the authors of this impressive earlier work for our ignorance about it in the first version of this paper posted on the archive. We also thank Jaume Garriga and Thomas Hertog for bringing this work to our attention.

## Introduction

One of the great achievements of modern cosmology is the ability to calculate detailed predictions for the cosmological perturbations from specific models of the early universe. This, along with impressive new data such as the WMAP survey[11] has allowed significant constraints to be placed on early universe physics as well as on a number of cosmological parameters.

One of the key cosmological parameters is  $\Omega_0$ , the ratio of the current cosmic density (including the dark energy) to the critical density. A well-known problem is that for

cosmological models with  $\Omega_0 < 1$  the perturbation calculation is more problematic, particularly on large scales. This is because for typical models of cosmic inflation to make precise predictions for perturbations on all observed scales they must also predict  $\Omega_0 = 1$  to about one part in  $10^5$ . In the context of these models, to calculate the large scale perturbations in the  $\Omega_0 < 1$  case one must answer the question “what physics other than inflation determined the perturbations on the largest observable scales?”. This issue has been recognized since the first papers on inflation with  $\Omega_0 < 1$ [12, 13].

For the most part, the cosmology community has “resolved” this problem by simply assuming a particular formula for perturbations in cosmologies with  $\Omega_0 < 1$ . This formula appears in all the main software packages (such as CMBfast) which determine the perturbation spectra for  $\Omega_0 < 1$  models. It is only because of this particular choice that it even seems possible to determine  $\Omega_0$  to high precision. One is left open to the possibility that a deeper understanding of early universe physics could shift our preference to different pictures of  $\Omega_0 < 1$  cosmology which could yield different formulas for the perturbation spectrum. For  $\Omega_0 < 1$  models with different spectra, the same data might well lead to a different preferred value of  $\Omega_0$  as well as other parameters.

In fact, we may be in the midst of such a shift right now. Recent work [14] suggests that string theory (our best hope for a realistic quantum gravity theory) predicts a landscape of different “vacua” which are highly stable, but which have some non-zero probability of tunneling into one another. This picture suggests a cosmology strikingly similar to the so-called “Open Inflation” models of Bucher et al. [15, 16].

The Open Inflation models were first invented to address the ambiguities of the perturbation spectra for  $\Omega_0 < 1$  cosmologies discussed above. Bucher et al. con-

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sider a cosmological model with an initial phase of inflation that defines the cosmological state on a range of length scales that spans many orders of magnitude and drives the global state of the universe toward  $\Omega_0 = 1$ . Bucher et al. modeled this phase of inflation with a field trapped in false vacuum, in the manner of “old inflation”[17].

This initial period of inflation ends with a tunneling process that produces a bubble universe which is open from the point of view of observers within it. The field that tunnels can experience a shorter period of slow-roll inflation[18] after the tunneling event which can bring the bubble universe close to  $\Omega = 1$  and define the perturbation spectrum on smaller scales. Because of the early period of old inflation the pre-tunneling cosmic state is uniquely determined, and this allows the perturbations in the bubble universe to be well determined on *all* observable scales with no ambiguities.

When first introduced the open inflation models seemed a bit artificial (although it really was a matter of taste whether one considered them more so than “typical” slow-roll inflation models). Today, the landscape picture that is emerging from string theory suggests that the cosmology for a universe in any one of the many metastable vacua universally starts with a tunneling event preceded by a long period of old inflation in the (false) vacuum of the previous landscape location. Although there still are a number of unresolved questions, this picture certainly suggests that the Open Inflation model of Bucher et al. may well be *the* universal cosmology seen by an observer in the string theory landscape.[35]

Our main motivation is the string theory landscape, but we also note that the puzzling low quadrupole and octopole ( $C_2$  and  $C_3$ ) in the WMAP first year data suggest that interesting information might be lurking in the cosmic perturbations on large scales[31, 32]. Since the open inflation perturbation spectrum depends not only on  $\Omega_0$ , but on the curvature of the inflaton potential during the period of old inflation (before tunneling) in principle we could read information about the physics of the universe before the big bang from large scale cosmological data.

With these motivations, we have undertaken a calculation of the CMB temperature anisotropies in Open Inflation models. Unfortunately, our results show that the differences between the open inflation results and the generic formula used in most cosmology papers is immeasurably small for realistic cosmological parameters. Thus we have nothing new to add to the interpretation of cosmological data. In particular, at least as far as the Open Universe models go, the standard determination of the value of  $\Omega_0$  and other cosmological parameters is unaltered, and there is no opportunity to measure new parameters from other parts of the string theory landscape. Of course this also means that we cannot rule out open inflation models with realistic values of  $\Omega_0$ . The one caveat is that our work assumes that the thin wall approximation gives a valid treatment of the tunneling

event. It is possible that corrections to this approximation could lead to a more interesting result.

It is also possible that a deeper understanding of the string theory landscape could lead to other kinds of predictive power in connection with open inflation. For example, a “most likely” form for the inflaton driving the post-tunneling period of new inflation could emerge, which in turn could lead to specific signature in the CMB power. This is not the effect we consider in this paper, which is devoted to effects generic to *all* open inflation models.

## I. THE PRIMORDIAL POWER SPECTRUM

The primordial power spectrum for Open Inflation presented in [16] is

$$P_\chi(\beta) = \frac{9}{4\pi^2} \left( \frac{H^3}{V_{,\phi}} \right)^2 \frac{1}{\beta(\beta^2 + 1)} \times \left[ \frac{e^{\pi\beta} + e^{-\pi\beta} + \frac{|C_2|}{C_1} \left( \frac{\beta+i}{\beta-i} e^{i\varphi} + \frac{\beta-i}{\beta+i} e^{-i\varphi} \right)}{e^{\pi\beta} - e^{-\pi\beta}} \right] \quad (1)$$

where  $k$  is the co-moving wavenumber which is related to  $\beta$  and the curvature  $K$  by  $k^2 = \beta^2 - K$ . With the usual normalization,  $k^2 = \beta^2 + 1$ . The field variable that gives the density fluctuations is  $\chi$ , and  $C_1$  and  $C_2$  are parameterized by

$$C_1 = 2\pi \cosh^2[\bar{\xi}(\beta)] \quad (2)$$

$$C_2 = 2\pi \cosh[\bar{\xi}(\beta)] \sinh[\bar{\xi}(\beta)] e^{i\varphi} \quad (3)$$

The definitions of  $C_1$  and  $C_2$  are

$$C_1 = 2\pi \left[ 1 + \frac{\sin^2(\pi\nu')}{\sinh^2(\pi\beta)} \right] \quad (4)$$

$$C_2 = 2\pi \frac{\sin(\pi\nu')\Gamma(i\beta - \nu')\Gamma(1 - i\beta)}{\sinh^2(\pi\beta)\Gamma(-i\beta - \nu')\Gamma(1 + i\beta)} \times (\cosh(\pi\beta)\sin(\pi\nu') - i\sinh(\pi\beta)\cos(\pi\nu')) \quad (5)$$

with  $\nu' = \sqrt{\frac{9}{4} - m^2} - \frac{1}{2}$ . Here,  $m^2$  is the false vacuum effective mass squared (the second derivative of the potential during the false vacuum inflation) in plank mass units. It is then more direct to express the power spectrum as

$$P_\chi(\beta) = \frac{9}{4\pi^2} \left( \frac{H^3}{V_{,\phi}} \right)^2 \frac{1}{\beta(\beta^2 + 1)} \times \left[ \coth(\pi\beta) + \frac{(\beta^2 - 1)Re(C_2) - 2\beta Im(C_2)}{C_1(\beta^2 + 1)\sinh(\pi\beta)} \right] \quad (6)$$

In open inflation, rather than having  $k^2\chi$  relating to the density fluctuations, we have  $(\beta^2 - 4K)\chi$  so the primordial power spectrum will be

$$P(\beta) = \frac{(\beta^2 + 4)^2}{\beta(\beta^2 + 1)} \times \left[ \coth(\pi\beta) + \frac{(\beta^2 - 1)Re(C_2) - 2\beta Im(C_2)}{C_1(\beta^2 + 1) \sinh(\pi\beta)} \right] \quad (7)$$

compared to the standard [33]

$$P(\beta) = \frac{(\beta^2 + 4)^2}{\beta(\beta^2 + 1)} \quad (8)$$

Thus, all that is necessary to compare Open Inflation predictions with the standard results is to insert the bracketed term into the initial power spectrum in the cmbopen subroutine of CMBfast[34]. The bracketed term quickly approaches unity for  $\beta > 1$  (wavelengths smaller than the curvature scale), so it effects the very largest scales without changing anything on small scales. For concreteness we take a tilt of unity ( $n_s = 1$ ).

## II. EVALUATION

To evaluate the CMB anisotropies from Open Inflation, the program CMBfast [34] was used to calculate CMB temperature power spectra, and an expression for the bracketed term was inserted into the subroutine cmbopen. As illustrated in Fig. 1, we explored the dependence of the bracketed term on the false vacuum mass and found that it controlled an oscillation in the bracketed term with respect to  $\beta$ . We found that  $m^2 = 4.5M_p^2$  yielded the strongest suppression of power at small wave number (of interest because of the WMAP anomalies).

An important feature to note is that the open power spectrum tends toward zero at very small wave numbers without the bracketed term, leaving only a limited window of wavenumbers for which the bracketed term has any effect. The bracketed term does diverge for most choices of  $m^2$  as wavenumber goes to zero, but not fast enough to overcome the rest of the power spectrum. This limits the effect of increasing the curvature on the power spectrum. Also note that, for  $m^2 = 4.5M_p^2$ , the bracketed term does not appear to diverge, but rather tends toward zero with no oscillation.

A wide range of curvatures were tested, comparing power spectra obtained from identical parameters with and without the correction. A best fit for parameters with a prior on  $\Omega_{tot}$  not being readily available[36], the choice of the parameters for these trials is a bit arbitrary. However, the effect of Open Inflation should be independent of all but the curvature scale, and we are comparing spectra that differ only by the inclusion or exclusion of

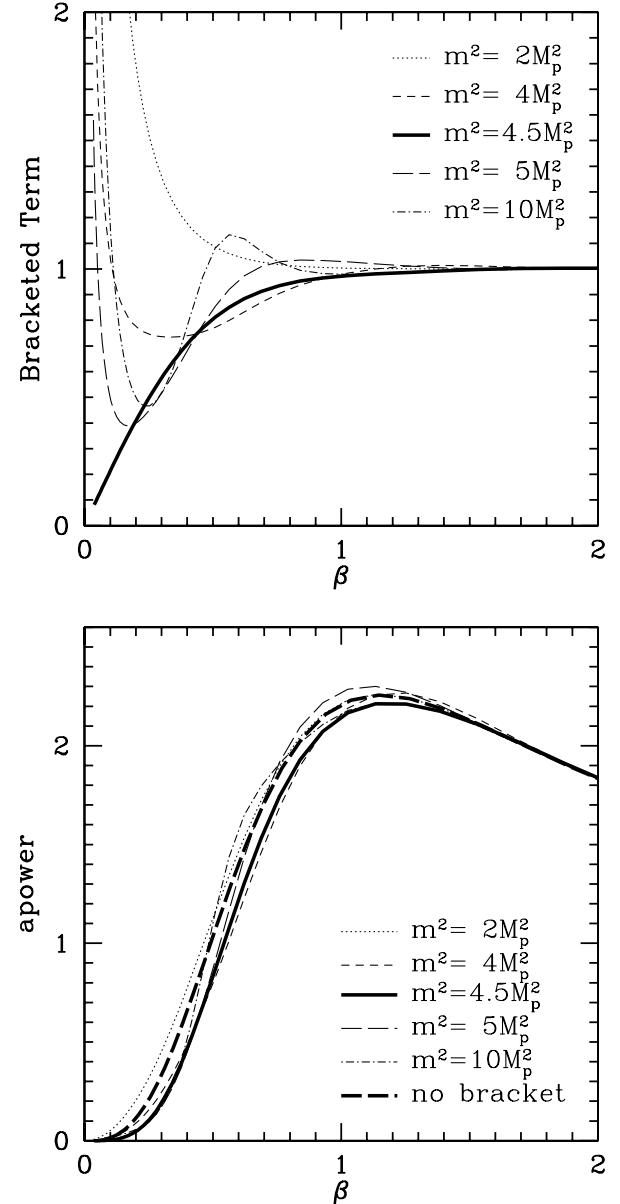


FIG. 1: Above is a plot of the bracketed term (top) and its effects on the open universe primordial power spectrum (bottom). As you can see, the different  $m^2$  values give the bracketed term different oscillations. The  $m^2 = 4.5$  plot is represented in each graph as the thick solid line; the thick dotted line is the power spectrum without the bracketed term.

the extra term that distinguishes Open Inflation. Given the small size of the difference the bracketed term generated, which are summarized in Table I, these concerns are largely unimportant.

$\Omega_{tot}$	% decrease in $C_2$
.99	$\sim .01\%$
.98	.02%
.95	.08%
.90	.20%
.85	.25%
.80	.27%

TABLE I: This table details the percent decrease in  $C_2$ , the  $l = 2$  value of power, caused by including the open inflation corrections with false vacuum mass  $m^2 = 4.5$  in plank units, as this value has the most effect on the primordial power spectrum. The first two entries were done using CMBfast with the best fit parameters given by the WMAP team, with the dark energy density reduced to achieve the stated total density. The rest were done simply using the CMBfast default settings with dark energy reduced. All had no re-ionization. We chose  $C_2$  to show here because the effect on the other multipoles was even less significant. The effect on the  $C_l$ 's is much less dramatic than on the quantities shown in the plots because the  $C_l$ 's depend on the power at many values of  $\beta$ , not just at  $\beta \approx 1$  where the effect on the power is most pronounced. The effect of the correction should depend on curvature and mass alone, so the values given will at least approximate those for any model with that curvature.

### III. CONCLUSIONS

The effects of Open Inflation on the CMB power spectrum are very small compared to the cosmic variance for the effected observables (Table 1 shows that the

most effected observable,  $C_2$ , experiences less than a 1% change, while it has a cosmic variance  $O(50\%)$ ). We conclude that the generic form for perturbations from Open Inflation are not distinguishable in the CMB temperature anisotropy power spectrum from perturbations given by the standard formula used throughout the literature. Thus this data cannot be used to identify evidence for or against Open Inflation or measure parameters in other vacua in the proposed string theory landscape that might be reflected in the Open Inflation primordial spectrum. Also, the general differences between the Open Inflation power spectrum and the standard version are so small that simply choosing between the two will not significantly impact constraints on cosmological parameters from CMB data. However, if our theoretical understanding evolves to the point where specific inflaton potentials are strongly preferred, a greater distinguishability between the two types of inflation might possibly emerge. We note that the power spectrum derived in [16] uses a thin wall approximation that may not be valid in many theories, and the effect of relaxing this assumption is unknown.

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Data available at <http://lambda.gsfc.nasa.gov>
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- [35] Others have considered inflation in the string theory landscape (see for example [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30]). Linde gives a nice review in sections 11 and 12 of [25]. For the most part these authors have sought a sufficiently large amount of new inflation at the onset of the big bang to make the issues raised here unimportant (that is, to make  $\Omega_0$  very close to unity). We have no argument against that approach, although in general it seems no easier to arrange potentials for long periods new inflation in the landscape picture than in other frameworks. The motivation for this paper is that models with slightly smaller values of  $\Omega_0$  might be considerably more interesting. In fact, any argument that the generic instance of inflation would tend to be short (as some have made in the landscape context) would favor a large effect of the sort we discuss here.
- [36] Given the overall disappointing nature of our results, it was not worth the effort to undertake a full-scale optimization in a large parameter space